Prove the derivative of $\tanh^{-1} x$ without using the logarithmic definition of $\tanh^{-1} x$. **SCORE: _____ / 4 PTS**

You may use the derivatives of the non-inverse hyperbolic functions that were listed in your textbook without proving them.

NOTE: You must prove the Pythagorean–like identity for $\tanh x$ if you wish to use it.

$y = \tanh^{-1} x$

$x = \tanh y$ (1/2)

$1 = \operatorname{sech}^2 y \frac{dy}{dx}$ (1)

$\frac{dy}{dx} = \frac{1}{\operatorname{sech}^2 y}$ (1/2)

$= \frac{1}{1-\tanh^2 y} = \frac{1}{1-x^2}$ (1/2)

$\cosh^2 y - \sinh^2 y = 1$

$\frac{\cosh^2 y - \sinh^2 y}{\cosh^2 y} = \frac{1}{\cosh^2 y}$ (1)

$1 - \tanh^2 y = \operatorname{sech}^2 y$ (1/2)

Find $\frac{d}{dx}\tanh^{-1}(\operatorname{sech} x)$.

SCORE: _____ / 3 PTS

You may use the hyperbolic identities and derivatives from your textbook without proving them.

(1) $\frac{1}{1-\operatorname{sech}^2 x} \cdot -\operatorname{sech} x \tanh x$ (1)

$= \frac{1}{\tanh^2 x} \cdot -\operatorname{sech} x \tanh x$

$= \frac{-\operatorname{sech} x}{\tanh x}$ (1/2) $= \frac{-\operatorname{sech} x}{\tanh x} \cdot \frac{\cosh x}{\cosh x} = -\frac{1}{\sinh x} = -\operatorname{csch} x$ (1/2)

Find $\lim_{x\to 0^-} \operatorname{csch} x$. **Do NOT use a graph. Give *BRIEF* algebraic or numerical reasoning.** **SCORE: _____ / 2 PTS**

$$\lim_{x\to 0^-} \frac{2}{e^x - e^{-x}} = -\infty \quad (1)$$

AS $x \to 0^-$, $e^x \to 1^-$ AND $e^{-x} \to 1^+$, SO $e^x - e^{-x} \to 0^-$ (1)

$$\frac{2}{0^-} \to -\infty$$

Write an algebraic expression to approximate the area under $f(x) = \ln x$ over the interval $[2, 7]$ using a left hand sum and n subintervals (as shown in class). **Do NOT evaluate the expression. Your answer must not use f ( ) notation.**

SCORE: _____ / 3 PTS

$$\sum_{i=0}^{n-1} f(a+i\Delta x)\Delta x$$

$a = 2$

$\Delta x = \frac{7-2}{n} = \frac{5}{n}$

$$= \sum_{i=0}^{n-1} f\left(2+\frac{5i}{n}\right)\frac{5}{n}$$

(1) (1) (1)

$$= \sum_{i=0}^{n-1} \frac{5}{n} \ln\left(2+\frac{5i}{n}\right)$$

OR

$$\ln(2)\cdot\frac{5}{n} + \ln\left(2+\frac{5}{n}\right)\cdot\frac{5}{n} + \ln\left(2+\frac{10}{n}\right)\cdot\frac{5}{n} + \ldots + \ln\left(2+\frac{5(n-1)}{n}\right)\cdot\frac{5}{n}$$

If $\coth x = -3$, find $\operatorname{sech} x$ and $\sinh x$.

SCORE: _____ / 5 PTS

$$\tanh x = \frac{1}{\coth x} = -\frac{1}{3} \quad (1)$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$1 - \left(-\frac{1}{3}\right)^2 = \operatorname{sech}^2 x \quad (1)$$

$$\operatorname{sech}^2 x = \frac{8}{9}$$

$$\operatorname{sech} x = \frac{2\sqrt{2}}{3} \quad (1)$$

SINCE $\operatorname{sech} x > 0$ FOR ALL x

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{3}{2\sqrt{2}}$$

$$(1) = \frac{3\sqrt{2}}{4}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

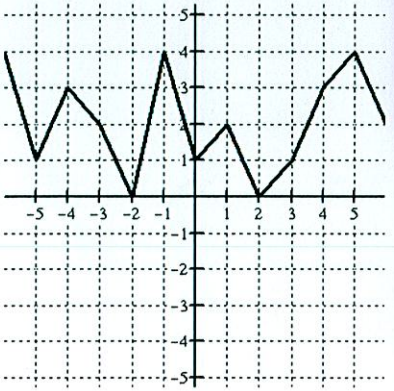
$$\sinh x = \tanh x \cosh x$$

$$= -\frac{1}{3} \cdot \frac{3\sqrt{2}}{4}$$

$$= -\frac{\sqrt{2}}{4} \quad (1)$$

Estimate the area under the function shown on the right over the interval $[-6, 2]$ using the left hand sum with 4 equal width subintervals.

SCORE: _____ / 3 PTS



$\Delta x = \frac{2 - -6}{4} = 2$ (1/2)

INTERVALS: $[-6,-4]\ [-4,-2]\ [-2,0]\ [0,2]$

AREA $\approx f(-6)\cdot 2 + f(-4)\cdot 2 + f(-2)\cdot 2 + f(0)\cdot 2$

$= \underbrace{4\cdot 2}_{\frac{1}{2}} + \underbrace{3\cdot 2}_{\frac{1}{2}} + \underbrace{0\cdot 2}_{\frac{1}{2}} + \underbrace{1\cdot 2}_{\frac{1}{2}}$

$= 16$ (1/2)